Graphing the conic with polar equation $r = \dfrac{a}{b + c\cos\theta}$ or $r = \dfrac{a}{b - c\cos\theta}$ or $r = \dfrac{a}{b + c\sin\theta}$ or $r = \dfrac{a}{b - c\sin\theta}$

All polar equations of the above four types correspond to conics with the pole as a/the focus

[1] Multiply numerator and denominator of the polar equation by $\frac{1}{b}$ to get a constant term of 1 in the denominator

The equation then becomes $r = \dfrac{A}{1 + e\cos\theta}$ or $r = \dfrac{A}{1 - e\cos\theta}$ or $r = \dfrac{A}{1 + e\sin\theta}$ or $r = \dfrac{A}{1 - e\sin\theta}$

[2] The eccentricity (e) is the absolute value of the coefficient of the trigonometric function in the denominator.
If $e = 1$, the conic is a parabola.
If $0 < e < 1$, the conic is an ellipse.
If $e > 1$, the conic is a hyperbola.

The numerator (A) is the eccentricity (e) multiplied by the distance from the pole/focus to the directrix (p).

$A = ep$, so $p = \dfrac{A}{e}$.

If the equation involves $\cos\theta$ in the denominator, then the directrix is vertical ($x = p$).
If the equation involves $\sin\theta$ in the denominator, then the directrix is horizontal ($y = p$).

If the coefficient of the trigonometric function in the denominator is positive,
the directrix is to the right of ($x = p$) or above ($y = p$) the pole/focus.
If the coefficient of the trigonometric function in the denominator is negative,
the directrix is to the left of ($x = -p$) or below ($y = -p$) the pole/focus.
The directrix is <u>NEVER</u> an axis of symmetry.

Part of the conic lies between the pole/focus and the directrix.
That part of the conic always curves around the pole/focus away from the directrix.

[3] Plot the points corresponding to $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$.

These are the $x-$ and $y-$intercepts of the conic.
NOTE: If the conic is a parabola, one of the four points will <u>NOT</u> exist.

The latus rectum passes through the pole/focus, and connects the two points above which are reflections of each other through the pole/focus.
That is, the two points whose rectangular co-ordinates are negatives of each other.
The other point (points) is (are) the vertex (vertices).

If the conic is an ellipse or a hyperbola:

[4] The center is the midpoint of the vertices. **The pole/focus is <u>NEVER</u> the center.**

[5] The center is also the midpoint of the two foci.
Double the co-ordinates of the center to get the other focus.

[6] The other latus rectum passes through the other focus and is symmetric to the first latus rectum.
The ends of the other latus rectum share a non-zero co-ordinate with the other focus,
and a non-zero co-ordinate with the ends of the first latus rectum.

[7] Use the vertices and the ends of the latera recta to sketch the conic.

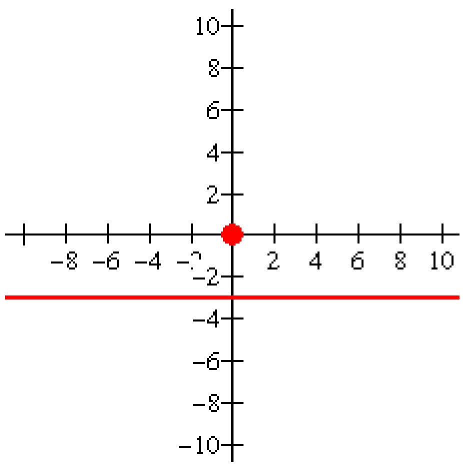
Example: Graphing the conic with polar equation $r = \dfrac{12}{2 - 4\sin\theta}$

[1] $$r = \frac{12}{2-4\sin\theta} \times \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{6}{1-2\sin\theta}$$

[2] $e = |-2| = 2 > 1$ ➡ hyperbola

$6 = ep = 2p$, so $p = 3$.

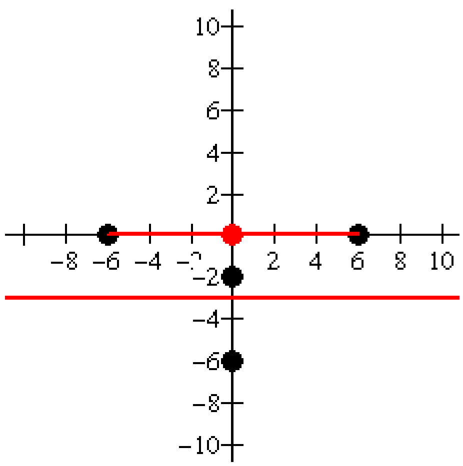
Since the equation involves $\sin\theta$ in the denominator,
and the coefficient of $\sin\theta$ in the denominator is negative,
therefore the directrix is horizontal and below the pole/focus at $y = -3$.



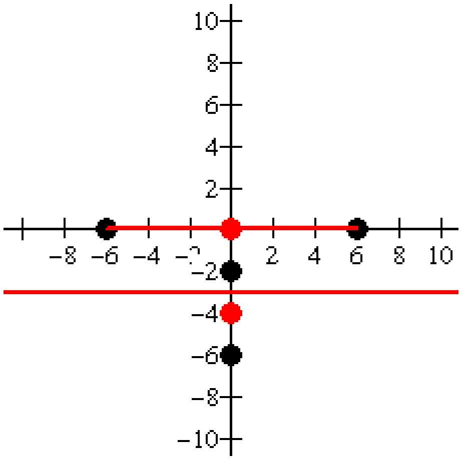
[3]

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$r = \frac{12}{2-4\sin\theta}$	6	-6	6	2
(x, y)	$(6, 0)$	$(0, -6)$	$(-6, 0)$	$(0, -2)$

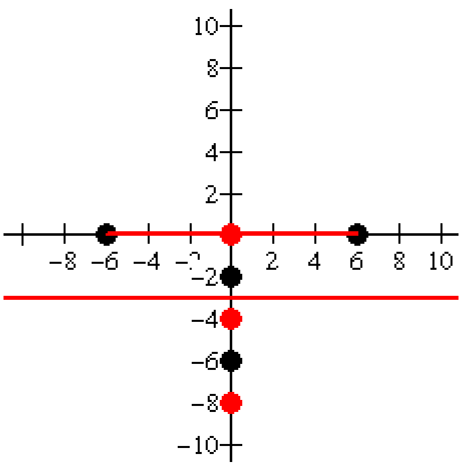
The latus rectum connects $(6, 0)$ and $(-6, 0)$.
The vertices are $(0, -6)$ and $(0, -2)$.



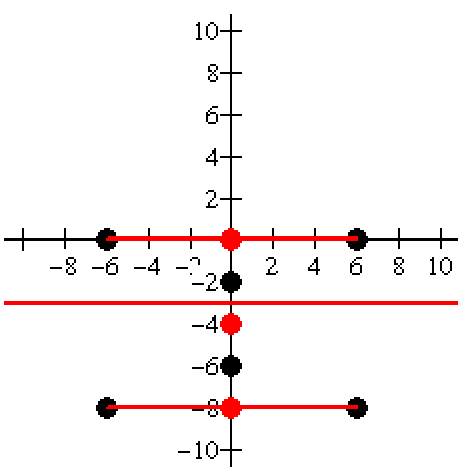
[4] The center is $\left(\frac{0+0}{2}, \frac{-6+-2}{2}\right) = (0, -4)$.



[5] The other focus is $(2\times 0, 2\times -4) = (0, -8)$.



[6] The other latus rectum passes through $(0, -8)$, $(6, -8)$ and $(-6, -8)$.



[7] Final result